

Geometry I, 2006 : Coursework 5

Show your calculations.

1. (i) [2 marks] Determine the intersection of the plane defined by $2x - 3y + 6z = 4$ with the line ℓ through the points $(1, 2, 3)$ and $(4, 5, 6)$.

(ii) [2 marks] Determine the intersection of the line ℓ of (i) with the line m through the point $(-2, 1, 1)$ in the direction of the vector $\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$.

2. [3 marks] Prove (without using co-ordinates) that $\underline{u} \times (\alpha \underline{v}) = \alpha(\underline{u} \times \underline{v})$ for all vectors $\underline{u}, \underline{v}$ and scalars α . [Hint: Read and understand the (revised) proof in the online lecture notes that $(\alpha \underline{u}) \times \underline{v} = \alpha(\underline{u} \times \underline{v})$.]

3. [4 marks] Simplify the following expressions (where $\underline{i}, \underline{j}, \underline{k}$ is a right-handed triple of pairwise orthogonal unit vectors).

(i) $(\underline{i} \times \underline{i}) \times \underline{j}$

(ii) $\underline{i} \times (\underline{i} \times \underline{j})$

(iii) $(\underline{k} \times \underline{j}) \times (\underline{j} \times \underline{i})$

(iv) $(\underline{i} \times \underline{j}) \cdot \underline{k}$

4. [2 marks] Find the distance from the point $(-1, 1, 3)$ to the line defined by Cartesian equations

$$\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-3}{4}.$$

5. (i) [2 marks] Find the distance between the lines defined by $\underline{r} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$

and $\underline{r} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

(ii) [2 marks] Find the distance between the lines defined by $\underline{r} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$ and

$$\underline{r} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 6 \\ -8 \end{pmatrix}.$$

6. [3 marks] Find a Cartesian equation for the plane through $(1, 2, 1)$ and parallel to the plane through $(1, -2, 3)$, $(-5, 0, 1)$ and $(2, 1, -1)$.

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 2nd November.

Leonard Soicher, 26 October 2006